

Bellwork: 5/6/13

Simplify each expression:

1) $(2x^3y^2z) \cdot (3x^5y^2z^9)$

$$6x^8y^4z^{10}$$

$$x^6 \cdot x^3 = x^9$$

2) $(3x^6yz^2)^3$

$$27x^{18}y^3z^6$$

$$(x^6)^3 = x^{18}$$

$$x^6 \cdot x^6 \cdot x^6 = x^{18}$$

Rational Exponents Notes

Objective: To evaluate expressions involving rational exponents.

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

Definition of Rational Exponents: If b is a real number and n is a positive integer such that the principal n th root of b exists, then $b^{\frac{1}{n}}$ is defined to be

$$\sqrt[n]{x} = x^{\frac{1}{n}} \quad \sqrt[3]{x^2} = x^{\frac{2}{3}} \quad b^{\frac{1}{n}} = \sqrt[n]{b}$$

$$\sqrt[5]{5x^4y^3z^2} = 5^{\frac{1}{5}}x^{\frac{4}{5}}y^{\frac{3}{5}}z^{\frac{2}{5}}$$

Also, if m is a positive integer that has no common factor with n , then

$$b^{\frac{m}{n}} = \left(b^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{b}\right)^m \quad \text{and} \quad b^{\frac{m}{n}} = \left(b^m\right)^{\frac{1}{n}} = \sqrt[n]{b^m}$$

$$x^{\frac{3}{2}} = (x^{\frac{1}{2}})^3 = (\sqrt{x})^3 \quad \text{or} \quad \sqrt{x^3}$$

The numerator of a rational exponent denotes the **power** to which the base is raised, and the denominator denotes the **index** or the **root** to be taken, as shown below.

$$b^{\frac{m}{n}} = \overbrace{\left(\sqrt[n]{b}\right)^m}^{\text{Power}} = \sqrt[n]{b^m}$$

When working with rational exponents, the properties of integer exponents still apply.

Changing from Radical to Exponential Form:

$$1) \sqrt[2]{3} = 3^{\frac{1}{2}}$$

$$2) \sqrt[2]{(3xy)^5} = (3xy)^{\frac{5}{2}} \\ 3^{\frac{5}{2}} x^{\frac{5}{2}} y^{\frac{5}{2}}$$

$$3) \sqrt[4]{x^3} = x^{\frac{3}{4}}$$

$$4) x^2 \cdot \sqrt[3]{x} = \boxed{x^{\frac{7}{3}}}$$

Calc: $2 + (1/3)$
 Com.den: $\frac{2}{1} + \frac{1}{3}$
 $\frac{2}{3/6} + \frac{1}{3} = \frac{7}{3}$

mixed #
 $2\frac{1}{3} = \frac{7}{3}$

Changing from Exponential to Radical Form:

$$1) x^{\frac{3}{5}} = \sqrt[5]{x^3}$$

$$2) 7^{\frac{1}{4}} = \sqrt[4]{7}$$

$$3) 2\sqrt[4]{x^3y} = 2x^{\frac{3}{4}}y^{\frac{1}{4}}$$

$$4) \frac{a^{\frac{3}{2}}}{1} = \frac{1}{a^{\frac{3}{2}}} = \frac{1}{\sqrt{a^3}}$$

$$5) x^{\frac{1}{3}}y^{\frac{1}{6}} = \sqrt[6]{x^2y}$$

Evaluating Expressions Involving Rational Exponents:

1) $27^{\frac{1}{3}} = \sqrt[3]{27}$

~~27^{\frac{1}{3}} = \sqrt[3]{27}~~

$27^{\frac{1}{3}} = \boxed{3}$

2) $25^{\frac{3}{2}} = \sqrt[3]{25^3} \text{ or } (\sqrt{25})^3$

$\boxed{125}$

3) $100^{-\frac{1}{2}} =$

$\frac{1}{100^{\frac{1}{2}}} = \frac{1}{\sqrt{100}}$

$= \boxed{\frac{1}{10}}$

4) $\left(\frac{16}{81}\right)^{\frac{1}{4}} =$

$\frac{16^{-\frac{1}{4}}}{81^{-\frac{1}{4}}} = \frac{81^{\frac{1}{4}}}{16^{\frac{1}{4}}} = \frac{\sqrt[4]{81}}{\sqrt[4]{16}}$

$\boxed{\frac{3}{2}}$

Worksheet 1 - Rational Exponents

Express using rational exponents. get rid of $\sqrt{\quad}$

1) $\sqrt[3]{5} =$

2) $\sqrt[5]{x^2} =$

3) $\sqrt[3]{c} = c^{\frac{1}{3}}$

4) $\sqrt[4]{3x^3} =$

5) $-\sqrt[5]{5^2} =$

6) $\sqrt{2x^3y^7} =$

Express in radical form. put in $\sqrt{\quad}$

7) $x^{\frac{3}{5}} =$

8) $7^{\frac{2}{3}} =$

9) $m^{\frac{1}{6}}n^{\frac{5}{6}} =$

10) $11^{\frac{1}{2}} =$

11) $(3x)^{\frac{2}{7}}y^{\frac{5}{7}} =$

12) $13^{\frac{1}{4}}x^{\frac{3}{4}} =$

Evaluate each of the following.

13) $32^{\frac{1}{5}} =$

14) $\left(-\frac{1}{64}\right)^{\frac{1}{3}} = (-1/64)^{\wedge(1/3)}$

15) $100^{\frac{3}{2}} =$

16) $243^{\frac{3}{5}} =$

17) $8^{\frac{5}{3}} =$

18) $-(144)^{\frac{1}{2}} = -(144)^{\wedge(1/2)}$

19) $32^{\frac{3}{5}} =$

20) $\left(\frac{27}{8}\right)^{-\frac{1}{3}} =$

21) $\left(\frac{36}{25}\right)^{\frac{1}{2}} =$

22) $81^{\frac{1}{4}} =$

